Related Rates

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How to solve related rates problems

1) **Draw a picture!**, labeling a couple of variables. **HOWEVER** do not put any numbers on your picture, except for constants! (otherwise you'll get confused later on)

2) Figure out what you *ultimately* want to calculate, and don't lose track of it

3) Find an equation relating your variables (perhaps not all of them)

4) Differentiate your equation using the chain rule/implicit differentiation.

5) **NOW** plug in all the numbers you know! Sometimes, you might need to calculate a number of 'missing variables'. Here an extra picture with all the numbers plugged in might be useful

6) Solve for whatever you were looking for in 2)

List of tricks

- Pythagorean theorem
- Formulas for areas and/or volumes
- Law of similar triangles
- Definition of sin and cos
- Law of sines, law of cosines

Problem 1

Suppose the volume of a sphere is increasing at a rate of $1cm^3/s$. At what rate is its radius increasing if $r = 2$ cm?

Problem 2

A cylindrical gob of goo is undergoing a transformation in which its height is decreasing at a rate of 1 cm/s while its volume is decreasing at the rate of 2π cm^3/s (It retains its cylindrical shape while all of this is happening). If, at a given instant, its volume is 24π cm^3 and its height is 6 cm, determine whether its radius is increasing or decreasing at that instant, and at what rate.

Problem 3

[3.9.15] Two cars start at the same point. Car A travels South at a rate of 6 mi/h and Car B travels West at a rate of 2.5 mi/h. At what rate is the distance between the two cars increasing 2 hours later?

Problem 4

[3.9.24] A trough is $10ft$ long and its ends have the shape of isosceles triangles that are $3ft$ across the top and have a height of $1ft$. If the trough is being filled with water at a rate of $12ft^3/min$, how fast is the water level rising when the water is 6 inches deep?

Problem 5

[3.9.30] A ladder 10 feet long rests against a vertical wall. The bottom of the ladder slides away from the wall at a rate of 1 ft/s. How fast is the angle between the ladder and the ground changing when the bottom is 6 feet from the wall?

Problem 6

[3.9.35] Two sides of a triangle have lengths $12m$ and $15m$. The anle between them is increasing at a rate of $2°/min$. How fast is the length of the third side increasing when the angle between the sides of fixed length is $60°$